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Kink dynamics in a discrete Φ^4 chain with dissipation and external field

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Abstract. We investigate the kink dynamics in a discrete Φ^4 chain with dissipation and external field. The kink, dressed and parametrized by its position $X(t)$, is introduced as a distinct degree of freedom in the discrete chain. By using the projection operator method with Dirac's second class constraints, we show that the asymmetric kink motion is modulated by the Peierls–Nabarro (PN) force whose barrier is a decreasing function of the external field. It is also seen that the average value of the PN force is an increasing function of the external field. The dressing of the kink profile is analysed numerically. It results in a considerable increase of the PN force amplitude.

1. Introduction

Fundamental studies have been carried out to analyse the behaviour of solitons in the presence of perturbations. These perturbations, currently encountered in physical systems, are of various sources: impurities, dissipation, external field (of mechanical, electric or magnetic nature) and noise. Their inclusion into non-linear lattice problems always leads to non-linear and intractable partial differential equations. To overcome the non-integrability of the resulting models and to analyse the perturbation effects on the structural and dynamical behaviour of solitons, various techniques have been elaborated [1–5]. Each technique is dictated by the nature of the perturbations and the choice of parameters to analyse. In general, there appears a modulation of the structural and dynamical parameters such as velocity, mobility, soliton extension and amplitude or an interesting dynamical behaviour [5, 6]. However, one can also observe a creation of new degrees of freedom. In some cases, the symmetry of the substrate potential of the lattice, and consequently that of solitons, can be completely modified. This is seen in a Φ^4 chain with dissipation and constant external field, where there exists a stable asymmetric domain wall [7, 20].

In some of these perturbed systems, the solitons are obtained after application of the continuum approximation to a set of discrete differential equations describing the movement of particles. This is the case in discrete lattices. When the soliton width is comparable to the lattice spacing, new phenomena can occur owing to discreteness effects. Therefore, one needs a mathematical formalism that takes the lattice effects into account. In ideal discrete lattices (without perturbations), some Hamiltonian formalisms have been used and interesting results obtained such as the energy loss to phonons and pinning effects [8–19]. But, in real systems, we need a description of the discrete soliton motion that involves perturbations. The first studies on the subject were carried out numerically by Peyrard and Kruskal for the one-dimensional sine–Gordon model [14] and Pnevmatikos *et al* for the Φ^4

model [20]. The effects of thermal fluctuations were analysed by Kaup and Osman [21]. Recently, by using a variational principle, which includes the generalized forces (obtained from the principle of virtual work) associated with damping and driving force, Pouget *et al* have shown that kink location in a three-dimensional lattice (discrete in one dimension) can be approximately described by a normalized damped–driven sine–Gordon equation [22]. These studies have been extended to incorporate localized impurities [23, 24].

We pursue the study in this paper, where we give an analytic description of kink dynamics in a Φ^4 chain with dissipation and external field. The development follows a letter published recently by the authors [25] where we neglected the effects of lattice discreteness on kink structure (the dressings). Here, we give a description of discrete kink motion in an asymmetric Φ^4 lattice. We concentrate on the determination of the Peierls–Nabarro (PN) force, on the numerical analysis of the corrections of the kink profile and the resulting effects on kink generalized potential. The effects of the external field on the PN force and on the kink dressing are obtained.

The organization of the paper is as follows. In section 2, we present the model and the resulting kink excitations. Section 3 deals with the mathematical formalism used to analyse the discrete kink dynamics. New dynamical variables are introduced: the position of the kink and the discrete corrections of the continuum soliton profile. The projection operator method is used to derive the equation of motion of the new variables. It is found that the asymmetric kink experiences the well known Peierls–Nabarro force, whose barrier is a decreasing function of external field. In section 4, a numerical analysis is performed to estimate the amplitude of dressing and its effects on the dynamical properties (the PN potential and frequency) of the discrete lattice. It is found that the inclusion of dressing increases the amplitude of the PN force. The last section is devoted to the conclusion.

2. Model and kink soliton excitations

We consider a monatomic chain of particles of mass m and equally spaced by the lattice constant b. Each particle is subjected to the anharmonic crystalline Φ^4 potential, which has been extended to include an external constant field f. The resulting on-site potential of the ith particle is therefore

$$W(y_i) = -\tfrac{1}{2}Ay_i^2 + \tfrac{1}{4}By_i^4 - fy_i. \tag{2.1}$$

The parameters A (which is generally temperature-dependent) and B are assumed to be positive constants. The function y_i is the displacement of the ith particle away from its mean position $x_i = ib$. The external force f may be due to a mechanical stress or to an electric field if we are in the presence of charged particles. In this latter case, the force f and the electric field E are related through the equation

$$f = e^*E \tag{2.2}$$

where e^* is the coupling constant or the effective charge of each particle. The nearest-neighbour particles are harmonically coupled with the elastic strain coefficient C, so that the Lagrangian of the lossless chain is defined as

$$\mathcal{L} = \tfrac{1}{2}\sum_i m\dot{y}_i^2 - \tfrac{1}{2}C\sum_i(y_{i+1} - y_i)^2 - \sum_i W(y_i). \tag{2.3}$$

The dot over y_i (or the subscript t hereafter) stands for the derivative with respect to time t. The external field applied uniformly to all particles adds to the asymmetry of the substrate potential. Indeed, for an external field less than $f_{\max} = (2/3\sqrt{3})A^{3/2}B^{-1/2}$, the potential (2.1) has two asymmetric stable equilibrium positions (see figure 1) defined as

$$(y_i)_1 = (2/\sqrt{3})y_0\cos((\theta/3) + (2\pi/3)) \qquad (y_i)_2 = (2/\sqrt{3})y_0\cos(\theta/3) \tag{2.4a}$$

instead of the well known symmetric equilibrium sites $y_i = \pm y_0$ with $y_0 = (A/B)^{1/2}$. The unstable equilibrium position is

$$(y_i)_3 = (2/\sqrt{3})y_0\cos((\theta/3) + (4\pi/3)) \tag{2.4b}$$

with θ defined as

$$\cos\theta = (3\sqrt{3}/2)(B^{1/2}/A^{3/2})f. \tag{2.4c}$$

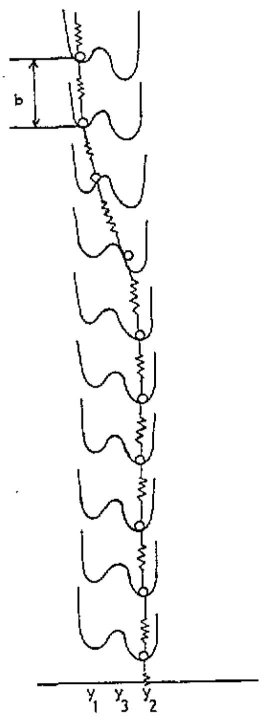

Figure 1. Structure of the chain of particles around the asymmetric kink with $f > 0$.

Since we deal with a dissipative chain, we need a Lagrangian description that involves dissipation. This can be achieved by adding a viscous damping term to the equation of motion resulting from the Lagrangian (2.3). One can also extend the Lagrangian formalism to include a Rayleigh dissipative function, which provides a phenomenological description of the frictional force with a damping coefficient λ.

From the Lagrangian (2.3), the equation of motion for the ith particle is (considering the damping)

$$m\ddot{y}_i + m\lambda\dot{y}_i - C(y_{i+1} + y_{i-1} - 2y_i) - Ay_i + By_i^3 - f = 0. \tag{2.5}$$

This set of discrete differential equations (i covers all the particles) is analytically intractable. It constitutes the basis of the projection operator method [26], which enables the analysis of kink dynamics in the discrete lattice (see section 3). In order to apply the projection method, one needs the kink solution of the continuum analogue of equation (2.5). This is obtained by replacing the discrete displacement $y_i(t)$ by the continuous displacement field $y(x, t)$ and the approximation

$$y_{i+1} + y_{i-1} - 2y_i \simeq b^2 y_{xx}$$

where the subscript xx (and tt hereafter) denotes the second spatial (and time) derivatives of the field $y(x, t)$. Then equation (2.5) becomes

$$my_{tt} + m\lambda y_t - mC_0^2 y_{xx} - Ay + By^3 - f = 0 \tag{2.6}$$

where $C_0^2 = Cb^2/m$ is the square of the speed of sound in the lattice.

When there is no damping and no external field ($\lambda = f = 0$), equation (2.6) reduces to the well known Φ^4 equation [27] whose topological soliton solution is

$$y(x, t) = \pm y_0 \tanh[(x - Vt)/L] \tag{2.7}$$

with the width L defined as

$$L = [2m(C_0^2 - V^2)/A]^{1/2} \tag{2.8}$$

where V is the constant velocity of kink (+) or antikink (−). The stability of this solution against small fluctuations has been established analytically [28] and numerically [29].

The effect of an external field applied in a lattice exhibiting a static or uniformly moving kink (2.7) has been analysed [14, 20, 30–32]. The main results are that there appears an adjustment of particles on the new equilibrium positions (2.4a). Since there is no damping ($\lambda = 0$), the kink accelerates continuously until it becomes so narrow that the discreteness effects cannot be neglected. Then, it starts emitting small-amplitude waves of specific frequencies. It thus reaches a limiting velocity V_f (less than the sound velocity) for which there is balance between the energy gained from the external field and the energy lost due to radiation of lattice phonons. Quantitative studies have shown that the limiting velocity V_f evolves by steps: for a large range of applied forces, V_f remains almost constant and then jumps to another value where it is again constant for a new range of f (see [14] for the sine–Gordon lattice and [32] for the Φ^4 lattice). Similar interesting results have also been obtained in [38] for two-component solitary waves where the evolution of the velocity versus the accelerating field and the damping coefficient presents an abrupt discontinuity and hysteresis phenomena.

When both the damping and the external field exist, equation (2.6) exhibits the bounded solution

$$y(x, t) = \pm y_0 \left(\eta_1 + \frac{\eta_2 - \eta_1}{1 + \exp[(x - Vt)/K]} \right) \tag{2.9}$$

corresponding to asymmetric domain walls [7] where

$$K = [2m(C_0^2 - V^2)/A]^{1/2}/(\eta_2 - \eta_1) \tag{2.10}$$

defines the asymmetric kink width. The coefficients η_1 and η_2 are the extreme zeros of the polynomial

$$P(\eta) = \eta - \eta^3 + f B^{1/2} A^{-3/2}. \tag{2.11}$$

They are given by the relations $\eta_1 = y_1/y_0$ and $\eta_2 = y_2/y_0$, where y_1 and y_2 are defined in equation (2.4a). Following [33], the constant velocity V, the damping coefficient λ and the coefficients η_1 and η_2 (i.e. the external field) are related by the equation

$$\lambda V[m/A(C_0^2 - V^2)]^{1/2} = \pm(3/\sqrt{2})\eta_3 \tag{2.12}$$

where η_3 defined as $\eta_3 = -(\eta_1 + \eta_2) = y_3/y_0$ is the third zero of the polynomial $P(\eta)$.

Assuming the non-relativistic limit, i.e. $V \ll C_0$ (which corresponds to the case of a high damping coefficient or small external field), the soliton width increases slightly with the force f (see figure 2).

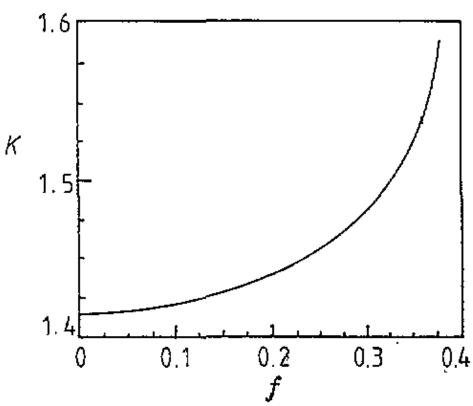

Figure 2. Slight increase of the asymmetric kink width K with the force f in the non-relativistic regime.

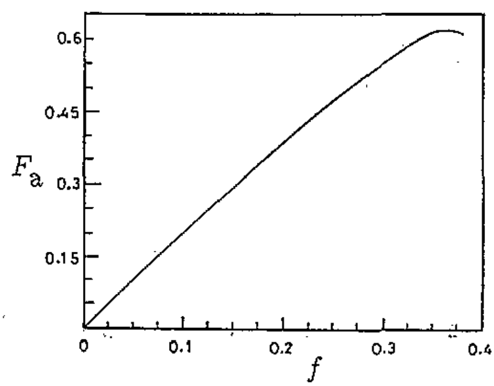

Figure 3. The average force F_a versus the external field f.

3. Asymmetric kink dynamics in the discrete chain

In this section, we are interested in the non-relativistic dynamics of the asymmetric kink (2.9) in the discrete chain. For this purpose, we consider the decomposition of the discrete displacement y_i in the following manner [25]:

$$y_i = y_K^i(X(t)) + \psi_i \tag{3.1a}$$

where the function $y_K^i(X(t))$ given by

$$y_K^i(X(t)) = -y_0\left(\eta_1 + \frac{\eta_2 - \eta_1}{1 + \exp\{[ib - X(t)]/K\}}\right) \tag{3.1b}$$

defines the continuum kink at the cell i ($x_i = ib$) of the lattice and the kink width has the non-relativistic form of equation (2.10), $X(t)$ stands for the collective coordinate or kink coordinate whose time behaviour is yet to be determined. In the continuum limit, $X(t)$ is proportional to time (e.g. $X(t) = Vt$, see equation (2.9)), but this is not the case in the discrete lattice, where the translational invariance of kink motion is broken by the periodic variation of kink parameters (see equation (3.4)).

From the decomposition (3.1a), employed in field theory [34], the kink is promoted to be a distinct degree of freedom through the collective coordinate $X(t)$. The $\psi_i(t)$ field accounts for the discrete correction or dressing of the first-order solution (3.1b) and for the radiated phonons emitted by the kink during its propagation. The introduction of the collective coordinate $X(t)$ adds two more degrees of freedom to the system corresponding to $X(t)$ and its conjugate momentum. Therefore, we need two constraints in order to conserve the original number of degrees of freedom. The choice of constraints must satisfy a double requirement: it should lead to an adequate canonical transformation and conserve the state of the physical system [25, 35]. The first constraint

$$C_1 = \sum_i y_K^{i(1)} \psi_i \simeq 0 \tag{3.2a}$$

minimizes the correction ψ_i in the vicinity of the kink centre. The superscript (1) on y_K^i denotes the derivative of y_K^i with respect to X. The second constraint, which ensures a functional link between $y_K^{i(1)}$ and $\dot{\psi}_i$, is defined as

$$C_2 = \sum_i y_K^{i(1)} \dot{\psi}_i \simeq 0. \tag{3.2b}$$

It maintains the quadratic form of the kinetic energy of the system by eliminating the cross-terms between the soliton kinetic energy and the particle kinetic energies. These constraints, originally used in the continuous field variables [34] and recently in the discrete lattice [15], are known as the second class constraints in Dirac's terminology [34, 35]. To derive the equation of motion for the new variables ψ_i, $\dot{\psi}_i$, X and $\dot{X}$, we will use the projection operator procedure [26] whose virtue is the ease with which the equations are derived relative to the amount of work needed in Dirac's formalism for constrained Hamiltonian dynamics [18, 19].

In that way, let us introduce the transformation (3.1a) into the basis equation (2.5). One obtains the equation of motion for the discrete correction ψ_i as

$$\begin{aligned} m\ddot{\psi}_i + m\lambda\dot{\psi}_i - C(y_K^{i+1} + y_K^{i-1} - 2y_K^i + \psi_{i+1} + \psi_{i-1} - 2\psi_i) - A(y_K^i + \psi_i) + B(y_K^i + \psi_i)^3 \\ - f + m\ddot{X} y_K^{i(1)} + m\lambda\dot{X} y_K^{i(1)} + m\dot{X}^2 y_K^{i(2)} = 0. \end{aligned} \tag{3.3}$$

In order to derive the equation of motion for the coordinate X, we 'project' equation (3.3) on $y_K^{i(1)}$. That is, we multiply equation (3.3) by $y_K^{i(1)}$ and then, taking the summation over all the particles, we make use of the constraining conditions to obtain

$$M\ddot{X} + M\lambda\dot{X} + \tfrac{1}{2}\dot{X}^2\, \mathrm{d}M/\mathrm{d}X + m\sum_i y_K^{i(1)} \ddot{\psi}_i = -\partial U/\partial X \tag{3.4}$$

with

$$M = \sum_i m(y_K^{i(1)})^2 \tag{3.5a}$$

standing for the kink mass and

$$\partial U/\partial X = \sum y_{\mathrm{K}}^{i(1)}[-C(y_{\mathrm{K}}^{i+1}+y_{\mathrm{K}}^{i-1}-2y_{\mathrm{K}}^{i}+\psi_{i+1}+\psi_{i-1}-2\psi_i)-A(y_{\mathrm{K}}^{i}+\psi_i)+B(y_{\mathrm{K}}^{i}+\psi_i)^3-f] \tag{3.5b}$$

is the partial derivative (with respect to X) of the generalized potential energy of the discrete chain. The set of equations (3.3) and (3.4) show that the discrete correction ψ_i is coupled to the collective coordinate X. The discreteness of the lattice gives rise to ψ_i that adiabatically dress the profile (3.1b), as well as to radiated phonons when the kink is set in movement. An accurate evaluation of the generalized force (3.5b) as functions of X and time, and consequently the time dependence of $X(t)$ and $\dot{X}(t)$, requires the determination of the discrete correction ψ_i. We concentrate hereafter on the determination of the potential force $\partial U/\partial X$ for a static asymmetric kink. We first neglect the correction ψ_i (see hereafter in this section), and in section 4 we take into account the dressing of kink structure.

In taking only the first-order approximation $y_i \simeq y_{\mathrm{K}}^{i}$, we use the Taylor expansion of y_{K}^{i+1} and y_{K}^{i-1} to the fourth-order derivative, and considering the static form of equation (2.6), we obtain that equation (3.5b) can take the form

$$\partial U/\partial X = \sum y_{\mathrm{K}}^{i(1)}[-Cy_{\mathrm{K}}^{i(2i)} - Ay_{\mathrm{K}}^{i} + B(y_{\mathrm{K}}^{i})^3 - f - (2C/4!)y_{\mathrm{K}}^{i(4i)}] \tag{3.6}$$

where the superscript (li) represents the lth derivative of y_{K}^{i} with respect to the discrete variable i. The right-hand side of (3.6) is a periodic function of X with period b (the lattice spacing). The Fourier coefficient of the summation with the fourth order of the derivative can be easily obtained, but for the term containing the y_{K}^{i} function we have used the computation to approximate its value. Therefore the PN force has the following form:

$$\partial U/\partial X = -\sum E_n \sin(2\pi nX/b) + F_{\mathrm{a}} \tag{3.7a}$$

with

$$E_n = \frac{2Cb(y_2-y_1)^2\pi^3 n^2}{180K^2\sinh(2\pi^2Kn/b)}\left(\frac{4\pi^2K^2n^2}{b^2}+1\right)\left(\frac{8\pi^2K^2n^2}{b^2}+3\right). \tag{3.7b}$$

The term F_{a} is an average force depending on f. It has been obtained numerically by inserting the function $y_{\mathrm{K}}^{i}(X)$ into equation (3.6) for a chain of $N = 200$ particles. The results presented in figure 3 show that F_{a} is an increasing function of f. As it later appears in figure 7, F_{a} increases when one takes into account the dressing ψ_i.

One can also find that the kink mass (see equation (3.5a)) has the following periodic structure:

$$M = M_0 + \sum M_n \cos(2\pi nX/b) \tag{3.8a}$$

with

$$M_0 = m(y_1-y_2)^2/6bK \tag{3.8b}$$

and

$$M_n = \frac{2m(y_1-y_2)\pi^2n^2}{3b^2\sinh(2\pi^2Kn/b)}\left(\frac{4\pi^2K^2n^2}{b^2}+1\right). \tag{3.8c}$$

Because of the presence of the hyperbolic sine function in the denominators of the Fourier coefficients, the contribution of second-order harmonics (or first-order ones for the mass) or higher can be neglected in comparison with the first-order one (or the fundamental M_0) (for instance, for $K = 1.3b$, it is seen that $E_2/E_1 \simeq 10^{-9}$).

The quantity $E_{\mathrm{PN}} = E_1/\pi$ can be seen as the Peierls–Nabarro potential amplitude (or barrier), which modulates the motion of dislocation in crystals [36]. It depends, as well as the kink mass, on the external field f through the terms $y_1 - y_2$. This dependence is an interesting result for the dynamical properties of kinks and other topological excitations (such as domain walls and dislocations). Indeed, figures 4 and 5 show the variations of E_1 and M_0 versus the external field. It is seen that the PN barrier decreases as the external field increases. This decrease is understandable if we appeal to the fact that the kink length increases with f. Moreover, this behaviour can be related to the lowering of the substrate potential barrier and consequently to the increase of the kink mobility.

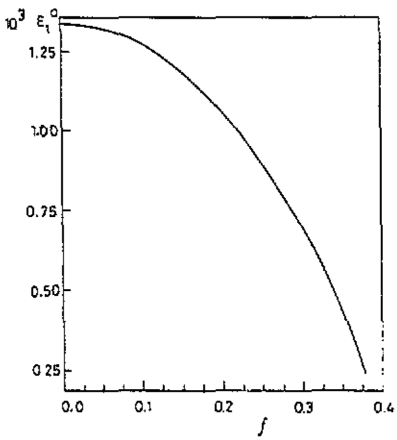

Figure 4. Barrier $E_1 \times 10^3$ versus the external field f for dimensionless parameters $A = B = 1$ and $C = 1$.

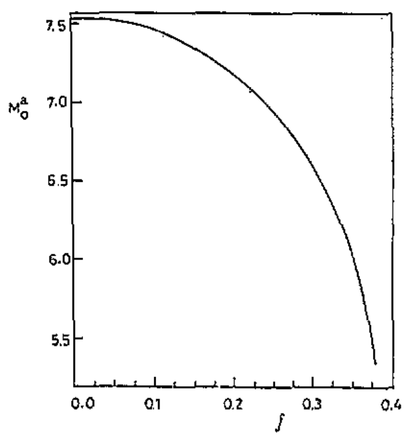

Figure 5. Mass M_0 versus the field f for dimensionless parameters $A = B = 1$ and $C = 1$.

As a result of equations (3.7), the velocity of an asymmetric kink has an oscillatory behaviour in the discrete lattice. With an insufficient kinetic energy, less than the PN barrier E_{PN}, the trapping of a kink can be observed. In this case, the kink oscillates about the bottom $X_k = (k + \frac{1}{2})b$ of the PN potential (k being an integer). The oscillatory frequency ω_{PN} defined as

$$\omega_{\mathrm{PN}} = (2\pi E_1/M_0)^{1/2} \tag{3.9}$$

is also a decreasing function of external field f.

When ψ_i is not neglected, the full equation (3.3) of the discrete correction has to be analysed. In one-dimensional or coupled sine–Gordon and Φ^4 discrete models, it has been shown that the ψ_i field has important effects on the dynamical properties, such as the kink pinning frequency ω_{PN} and the depth of the PN potential [11, 16, 19]. The analysis of phonon effects associated with the ψ_i field leads to a dissipative character for the kink motion in the discrete lattice [12, 14, 37]. Section 4 deals with the numerical analysis of the static form of equation (3.3). The effects of the dressing on the amplitude of the PN force are presented.

4. Numerical analysis of static dressing and its effects on the PN force

The discrete corrections ψ_i in the static form satisfy the set of non-linear discrete equations

$$-C(y_K^{i+1} + y_K^{i-1} - 2y_K^i + \psi_{i+1} + \psi_{i-1} - 2\psi_i) - A(y_K^i + \psi_i) + B(y_K^i + \psi_i)^3 - f = 0 \tag{4.1}$$

which can be obtained by minimizing the potential

$$U = \tfrac{1}{2}C\sum(y_K^{i+1} + \psi_{i+1} - y_K^i - \psi_i)^2 + \sum W(y_K^i + \psi_i) \tag{4.2}$$

with respect to ψ_i. Equation (4.1) describes the dressing for a kink located in one of the equilibrium sites of the PN potential. Assuming that the ψ_i fields are small enough to justify the linearization in equation (4.1), we obtain the following matrix equation:

$$\mathbf{A}\Psi = \mathbf{F} \tag{4.3}$$

where Ψ and $\mathbf{F}$ are column matrices defined respectively as

$$\Psi = \begin{bmatrix} \vdots \\ \psi_i \\ \vdots \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} & \vdots & \\ -C(y_K^{i+1} + y_K^{i-1} & -2y_K^i & -Ay_K^i + B(y_K^i)^3 - f \\ & \vdots & \end{bmatrix} \tag{4.4}$$

and $\mathbf{A}$ is a tridiagonal matrix with matrix elements $(\mathbf{A})_{ij}$ given by

$$(\mathbf{A})_{ij} = -C\delta_{i,j-1} + [2C - A + 3B(y_K^i)^2]\delta_{i,j} - C\delta_{i-1,j}. \tag{4.5}$$

For a kink situated at a non-equilibrium site of the PN potential, the analysis of dressing requires an extra force that holds the kink and prevents it from returning to the PN well. The required force can be generated from the Lagrange multiplier technique associated with constraint C_1 [16]. This is achieved by adding the constraint C_1 with an undetermined Lagrange multiplier $\alpha(X)$ to the original potential (4.2), so that the new potential has the form

$$U \longrightarrow U + \alpha(X)\sum y_K^{i(1)}\psi_i. \tag{4.6}$$

The procedure, also known as a quasistatic approach, introduces the restraining force that balances the PN force and maintains the kink in a non-equilibrium position. The minimization of the new potential (4.6) adds another inhomogeneous term to the right-hand side of equation (4.3), so that we have

$$\mathbf{A}\Psi = \mathbf{F} - \alpha(X)\mathbf{Y}_K^{(1)} \tag{4.7}$$

where $\mathbf{Y}_K^{(1)}$ is a column matrix defined by

$$\mathbf{Y}_K^{(1)} = \begin{bmatrix} \vdots \\ y_K^{i(1)} \\ \vdots \end{bmatrix}. \tag{4.8}$$

Solving equation (4.7) for $\alpha(X)$ leads to the expression

$$\alpha(X) = (\mathbf{Y}_K^{(1)})^T \mathbf{A}^{-1} \mathbf{F} [(\mathbf{Y}_K^{(1)})^T \mathbf{A}^{-1} \mathbf{Y}_K^{(1)}]^{-1} \tag{4.9}$$

where $\mathbf{A}^{-1}$ is the inverse matrix of $\mathbf{A}$ and $(\mathbf{Y}_K^{(1)})^T$ is the transpose of the column matrix $\mathbf{Y}_K^{(1)}$. The dressing is then obtained by substituting equation (4.9) into equation (4.7).

For the numerical calculation of the dressings, the solution of equations (4.3) and (4.7), we have considered a system of N particles with $N = 200$ and we have used the method presented in [19]. The computations have been carried out for varying values of the external field. The following results have been obtained.

We have observed that the maximum amplitude of the dressing decreases slightly when the force f increases; see figure 6 where the variations of the dressing versus the distance $i - X$ have been plotted for two different values of the external field. This result can be justified by the fact that the kink extension in the non-relativistic regime increases with the force f, thus reducing the lattice effects. It has also been seen that $(\psi_i)_{max}$ is less than 0.015, a small value that justifies the linearization operated on equation (4.1). The constraint C_1 is satisfied since ψ_i is an odd function of $i - X$ while $y_K^{i(1)}$ is an even function.

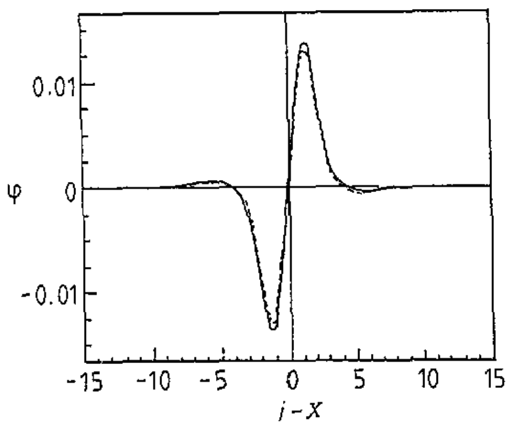

Figure 6. Variation of dressing as a function of the distance $i - X$ for $f = 0.01$ (full curve) and $f = 0.2$ (broken curve).

In figure 7, we have plotted, for $f = 0.1$, the PN force (obtained by numerical calculations for 200 lattice particles) versus X in two periods of the lattice. It appears that $\partial U/\partial X$ is periodic as obtained in analytic discussions. The amplitude of the PN force is seen to increase considerably when one includes the dressing ψ_i in calculating the PN force (compare the amplitude in figure 7(a) and that in figure 7(b)). Such behaviour due to the relaxations of the kink profile had earlier been obtained by Aubry [8].

5. Conclusion

In this paper, we have studied the dynamics of kinks in a discrete Φ^4 chain with dissipation and external field. After having reviewed the main topological soliton excitations in the continuum lattice, we have used the collective coordinate method in which the position $X(t)$ of the kink centre appears as an unknown dynamical variable. In addition, owing to the discreteness, the continuum kink profile has been dressed by the discrete correction ψ_i. The ψ_i field takes into account the dressing of the kink profile and the phonons radiated

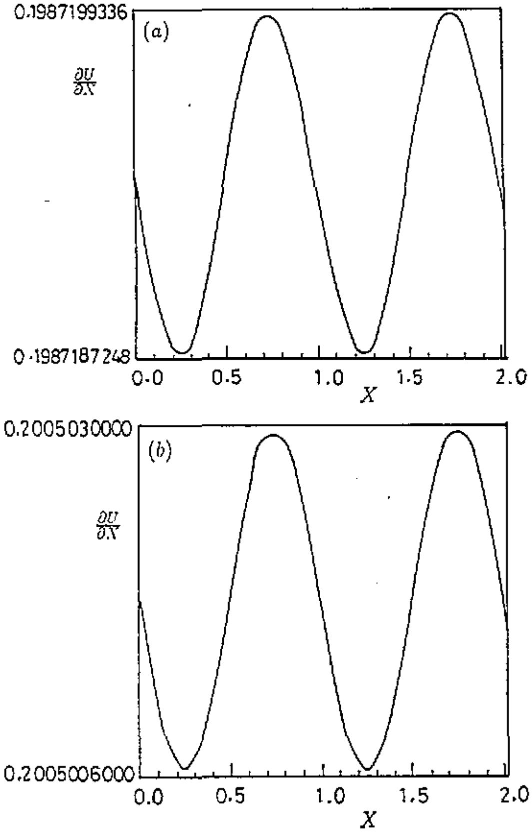

Figure 7. The Peierls–Nabarro force $\partial U/\partial X$ versus X in two lattice spacings: (a) without dressing and (b) with the contribution of dressing for $f = 0.1$.

by a propagating kink in the discrete lattice. Using two suitable constraining conditions, we have used the projection operator procedure to derive the equation of motion for the position $X(t)$. It has been shown that the kink propagation is modulated by a periodic PN potential whose barrier height decreases when the external field increases while the average value increases. The decaying of the PN barrier has been explained as due to the fact that the external field lowers the barrier of the substrate potential and therefore increases the kink mobility.

An equation for the discrete correction ψ_i has been obtained. We have numerically studied its static form and it appears that the amplitude of the dressing decreases when the external field increases. These results are understandable since the kink extension (in the non-relativistic regime) increases with the external field, thus reducing the lattice effects. The inclusion of dressing effects considerably improves the accuracy in estimates of dynamical quantities such as the pinning frequency and the amplitude of the PN force.

The results obtained in this paper can be used to discuss the diffusion of domain walls in ferroelectric materials such as SbSI and $Pb_5Ge_3O_{11}$. Moreover, the model can be used to describe proton diffusion in hydrogen-bonded materials (for example, in ice and in ferroelectrics such as KH_2PO_4) with, however, the inclusion of heavy-ion excitations (see [38, 39]).

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